

Learning to Teach Mathematics Using The Geometer's Sketchpad through Lesson Study

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Abstract

The purpose of this study was to help pre-service secondary teachers to teach mathematics using The Geometer's Sketchpad (GSP) through Lesson Study (LS) which was incorporated into a mathematics teaching methods course. LS is a Japanese model of teacher professional development in which small groups of teachers collaboratively plan, teach and revise a lesson to improve the quality of their teaching as well as to enrich students' learning experiences. The researcher employed a case study research design and the participants consisted of 26 pre-service secondary teachers who enrolled in the mathematics teaching methods course in a Malaysian public university. Thirteen LS groups each comprising 2 pre-service secondary teachers were set up in two tutorial groups with six LS groups in the first tutorial group and seven LS groups in the second tutorial group. Qualitative data were collected for each LS group through observations, written lesson plans, reflections and GSP sketches. This paper discusses the changes in the constructed GSP sketches of one of the LS groups for estimating the value of π in the first, second and third lessons. Findings of the study indicate that the participants of this LS group showed gradual improvement in their ability to construct GSP sketches for teaching the topic after engaging in LS.

Keywords: The Geometer's Sketchpad, Lesson Study, Pre-service secondary teachers

Background of the Study

GSP is a dynamic geometry software program for constructing and investigating mathematical objects. It is a dynamic tool for construction, demonstration and exploration that adds a powerful dimension to the teaching and learning of geometry and many other areas of mathematics. In fact, research has shown that GSP is an important tool for enhancing students' learning of mathematics. For example, Elchuck (1992) found that mathematics achievement and time of independent investigation using GSP were significant predictors of conjecture-making ability. Frerking (1995) found that the abilities to conjecture and justify conjectures in a geometry class using GSP were directly related to proof-writing abilities. In addition, the use of GSP enhanced: (a) students' van Hiele levels of geometric thinking (Choi, 1996; Choi-Koh, 1999; July, 2001; McClintock, Jiang & July, 2002; Thompson, 2006); (b) secondary students' geometry achievement and van Hiele levels of geometric thinking (Chew, 2007; Chew & Noraini Idris, 2006; Noraini Idris, 2007; Nurul Hidayah Lucy, 2005); (c) primary pupils' van Hiele levels of geometric thinking of selected regular polygons (Chew & Lim, 2010); and (d) pre-service secondary mathematics teachers' understanding of limits of sequences (Cory & Garofalo, 2011). Further, the dynamic capability of GSP, inquiry-based tasks, as well as student-student and researcher-student interactions deepened students' conception of two-dimensional shapes (Driskell, 2004). Moreover, Rosanini Mahmud, Mohd Arif Hj Ismail and Lim (2009) found that a GSP-based courseware called 'G-Reflect' had a significant effect on secondary students' achievement and motivation in learning the topic of Reflections.

In view of its importance, the Malaysian Ministry of Education (2003) advocates the use of GSP in the teaching and learning of mathematics by purchasing the GSP license and

supplying the GSP software to all secondary schools in Malaysia since 2004. While it is envisaged that this initiative would benefit many students, teachers and teacher educators nationwide, teacher enthusiasm and willingness to use GSP in the teaching and learning of mathematics in the classroom remains an issue to be addressed (Teoh & Fong, 2005). A survey conducted by Kasmawati (2006) on 151 secondary mathematics teachers in the state of Penang revealed that 26% of the teachers had attended GSP training courses but only 2% used GSP to teach mathematics in the classroom. The two main reasons given by the mathematics teachers were firstly lack of time to prepare GSP sketches, and secondly lack of skills and confidence to use GSP to teach mathematics in the classroom.

Hence, there is an urgent need to help pre-service secondary mathematics teachers' to teach mathematics using GSP by developing their skills and confidence to construct GSP sketches for teaching mathematics through a collaborative group effort such as Lesson Study which will provide assistance and support as well as sustain the continuous integration of GSP in the teaching and learning of mathematics in the classroom as advocated by the Malaysian Ministry of Education (2003).

Purpose of the Study

The purpose of this study was to help pre-service secondary teachers to teach mathematics using GSP through LS. Specifically, the focus of this paper was to examine the changes in the constructed GSP sketches of one of the LS groups for estimating the value of π in the first, second and third lessons after engaging in LS.

Conceptual Framework

The researcher employed Lesson Study (LS) as the conceptual framework underpinning this study. According to Fernandez and Yoshida (2004), LS is a direct translation for the Japanese term *jogyokenkyu*. *Jogyokenkyu* is composed of two words: *jogyo*, which means lesson, and *kenkyu*, which means study or research. Since the 1960s, LS was already well established in Japan and it is still an on-going practice as a form of teacher professional development whereby teachers actively engage in a continuous process of improving the quality of their teaching and to enrich their students' learning experiences as well. Basically, LS is a process by which small groups of teachers meet at stipulated time to plan lessons, observe these lessons unfold in actual classrooms, discuss their observations and to revise the lesson plans collaboratively. More specifically, LS comprises six main steps: (1) collaboratively planning the lesson plan, (2) seeing the lesson plan in action, (3) discussing the lesson plan, (4) revising the lesson plan, (5) teaching the new version of the lesson, and (6) sharing reflections about the new version of the lesson (Fernandez & Yoshida, 2004). These six steps are discussed in more detail in the research procedure section.

A number of studies have shown that LS improves teachers' learning and supports teachers to grow professionally as well (Fernandez, & Yoshida, 2004; Lewis, 2000; Lewis & Tsuchida, 1998; Lim, White & Chiew, 2005; Shimahara, 1998; Stigler & Hiebert, 1997, 1999; Yoshida, 1999). In addition, Chew and Lim (2011a) found that LS encouraged the innovative use of GSP in the teaching and learning of mathematics among secondary school teachers. The teachers showed positive changes in their knowledge and skills of using GSP to teach the topics of Lines and Planes in Three Dimensions, Loci in Two Dimensions as well as Plans and Elevations. They also showed positive acceptance and feedback about LS such as

providing peer support and collaboration. The teachers thus had more confidence in using GSP to teach mathematics at the secondary school level after engaging in LS.

Apart from that, several studies have also shown that LS provides worthwhile and beneficial learning experiences for pre-service teachers. Chiew and Lim (2003) found that LS improved pre-service mathematics teachers' content knowledge and enhanced their confidence to teach mathematics, and they gained much more diverse teaching ideas that helped them improve their pedagogical content knowledge as well. Fernandez and Robinson (2006) found that LS helped pre-service teachers to connect theory and practice, collaborate among themselves as well as reflect on their teaching and learning experiences. Lim (2006) found that despite facing problems of time constraint and peer conflict, most pre-service teachers suggested that LS was a good means of preparing them to teach mathematics and would like to continue to be involved in LS in schools after graduation. Moreover, Chew and Lim (2011b) showed that LS enhanced pre-service secondary teachers' skills of using GSP to teach the topic of Regular Polygons in the Malaysian Form Three Mathematics syllabus.

Methodology

Research design and participants

The researcher employed a case study research design and purposive sampling to select the participants of the study which consisted of twenty-six pre-service secondary teachers who enrolled in a mathematics teaching methods course in a Malaysian public university.

Research procedure

The research procedure of this study consisted of three main parts. The first part was the first two-hour lecture in which the researcher, who is the coordinator of the course, explained the outline of the course, the coursework, and LS to all the participants. At the end of the lecture, the participants were divided into two tutorial groups. Next, thirteen LS groups each comprising 2 participants were set up in the two tutorial groups. There were six LS groups in the first tutorial group (referred to as LS Group 1 to LS Group 6) and seven LS groups in the second tutorial group (referred to as LS Group 1 to LS Group 7). Each tutorial group would meet at a specific tutorial time for one hour every week.

The second part comprised two GSP workshops for each tutorial group conducted by the researcher during the first two tutorials. In the first GSP workshop which was held during the first tutorial, the participants learnt the functions of the Title bar, Menu bar, Sketch plane, and Toolbox of GSP as well as how to use the basic tools of GSP such as Selection Arrow tool, Point tool, Compass tool, Straightedge tool, Text tool, and Custom tool to construct mathematical objects like points, segments, rays, lines, circles, and polygons. In the second GSP workshop which was held during the second tutorial, the participants learnt how to construct a GSP sketch for teaching the topic of Pythagoras' Theorem based on Bennett's (1999) GSP activity sheet. After the workshops, the participants were encouraged to learn more about GSP by referring to GSP books which were available in the library and other resources on the Internet.

In the third part of the research procedure, the researcher implemented the six main steps of LS as outlined by Fernandez and Yoshida (2004) during the subsequent tutorials:

Step 1 (Collaboratively Planning the Lesson Plan):

During the third tutorial each LS group was allowed to choose a topic in the Malaysian secondary school mathematics syllabus and then collaboratively developed a 40-minute lesson plan for teaching the chosen topic with GSP. At the end of the tutorial, each LS

group had to plan subsequent meetings outside the lecture and tutorial schedule to complete their lesson plan and GSP sketches before the fourth tutorial.

Step 2 (Seeing the Lesson Plan in Action)

During the fourth tutorial one member from LS Group 1 (Teacher 1) in each tutorial group taught the 40-minute lesson as planned using GSP version 4.05M installed in his/her laptop and a mounted LCD projector to their peers in the Mathematics Teaching Room. The lesson was observed by his/her partner of LS Group 1 (Teacher 2) and the researcher.

Step 3 (Discussing the Lesson Plan)

After the lesson, the peers and the researcher provided comments and suggestions to improve the lesson plan and GSP sketches.

Step 4 (Revising the Lesson Plan)

After the tutorial, the members of LS Group 1 in each tutorial group planned subsequent meetings outside the lecture and tutorial schedule to revise their lesson plan and GSP sketches based on their peers' and the researcher's comments and suggestions as well as their own observations before the fifth tutorial.

Step 5 (Teaching the New Version of the Lesson)

During the fifth tutorial the new version of the lesson was then taught by the other partner of LS Group 1 (Teacher 2) using GSP version 4.05M installed in his/her laptop and a mounted LCD projector to different peers in the other tutorial group in the Mathematics Teaching Room. The lesson was observed by his/her partner of LS Group 1 who had taught the first lesson (Teacher 1) and the researcher. After the lesson, the peers and the coordinator provided comments and suggestions to further improve the lesson plan and GSP sketches.

Step 6 (Sharing Reflections about the New Version of the Lesson)

After the tutorial, the members of LS Group 1 in each tutorial group again planned subsequent meetings outside the lecture and tutorial schedule to revise their lesson plan and GSP sketches based on their peers' and the researcher's comments and suggestions as well as their own observations before the sixth tutorial. The end product of this last step would be a final lesson plan and GSP sketches for submission to the researcher as their coursework.

Steps 2 to 6 were repeated for the other LS Groups (that is LS Groups 2, 3, 4, 5 and 6 in the first tutorial group and LS Groups 2, 3, 4, 5, 6 and 7 in the second tutorial group) in the subsequent tutorials respectively. Qualitative data were collected for each LS group through observations, written lesson plans, reflections and GSP sketches.

Findings and Discussion

In this paper, the discussion focuses on the analysis of the GSP sketches in the first, second and third lessons of one of the LS groups. This LS group comprised two female participants. They chose to construct a GSP sketch for estimating the value of π in the Malaysian Form Two (the second year of secondary school) Mathematics syllabus. The changes in the participants' constructed GSP sketches in the first, second and third lessons after engaging in LS are presented and discussed in the following sections respectively.

GSP sketch in the first lesson

The learning objective of the first lesson was to enable students to estimate the value of π . Thus, during the first lesson, the first member of the LS group (Teacher 1): (1) constructed a circle using the Compass tool and labelled its centre and the point on its circumference as O and A respectively using the Text tool; (2) constructed radius OA using the Segment tool and measured its length using the Length option in the Measure menu; (3) calculated the length of the diameter of the circle by multiplying the length of the radius with 2 using the Calculate option in the Measure menu; (4) measured the circumference of the circle using the Circumference option in the Measure menu; and (5) calculated the ratio of the circumference to the diameter of the circle using the Calculate option in the Measure menu. The resulting GSP sketch is as shown in Figure 1.

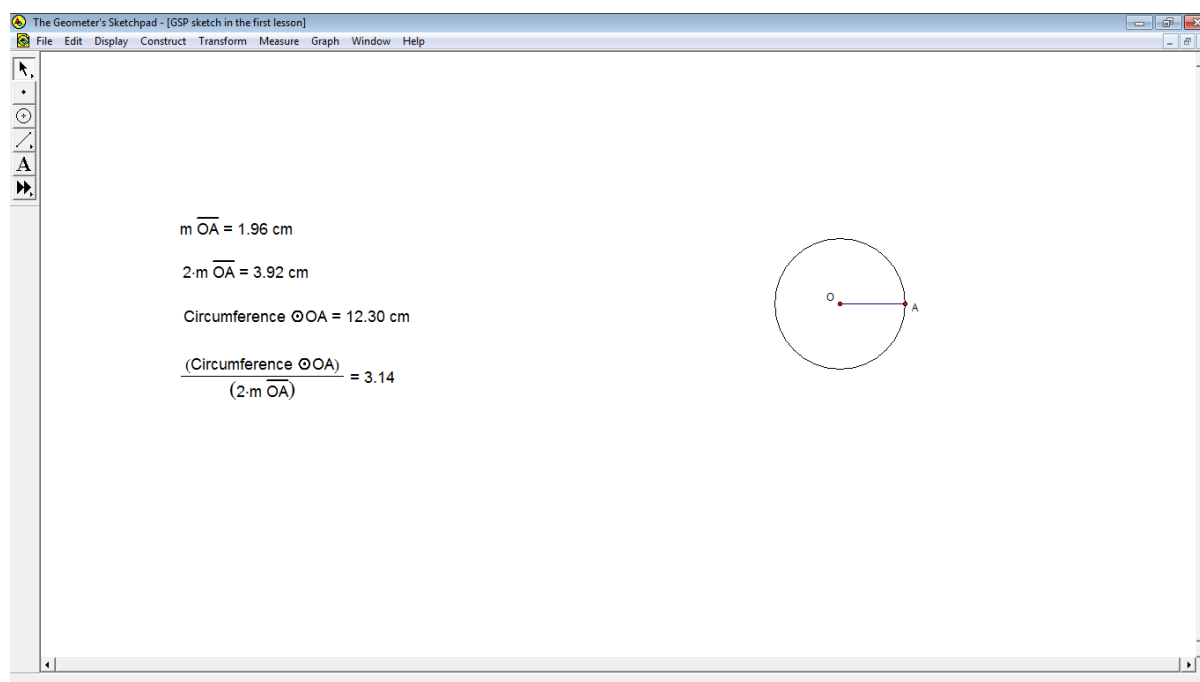


Figure 1. GSP sketch in the first lesson

Next, Teacher 1 dragged point A to change the size of the circle and asked the students (peers) to observe the changes in the values of all the measurements. Following this, the students were asked to draw a conclusion about the value of the ratio of the circumference to the diameter of a circle based on their observations. Finally, Teacher 1 explained that the ratio of the circumference to the diameter of a circle is known as π and its estimated value is 3.14 correct to two decimal places.

After the lesson the peers who were sitting at the back of the room commented that the circle, labels and measurements of the radius, diameter and circumference of the GSP sketch were too small to be seen clearly by them. Some of the peers also commented that students might not understand the symbols used by GSP to represent the radius, diameter and circumference of the circle because they are not being used in the Malaysian Form Two Mathematics textbook. Thus, they suggested that the symbols ought to be changed to help students better understand the measurements.

In addition, the researcher commented that the measurements of the circumference, diameter and the ratio of the circumference to the diameter of a circle ought to be at least in three decimal places so that the estimated value of π would also be at least in three decimal

places as used in the Malaysian Form Two Mathematics textbook. Apart from that, the researcher suggested that a diameter ought to be constructed and then measured its length using the Calculate option in the Measure menu instead of calculated the length of the diameter of the circle by multiplying the length of the radius with 2 using the Calculate option in the Measure menu. The researcher also suggested that a table ought to be constructed to show the changes in the values of the circumference and diameter but the value of the ratio of the circumference to the diameter of a circle remains constant as the size of the circle changes. This would help students to better understand the ratio of the circumference to the diameter of a circle as well as the concept of π and its estimated value.

After the tutorial, the members of the LS group had to make changes to their GSP sketch based on the comments and suggestions given by their peers and the researcher as well as their own observations. They did further readings on constructing GSP sketches for teaching the topic by referring to GSP books in the library and other resources on the Internet. They also consulted the researcher on how to enlarge the labels and measurements, change the labels as well as construct a table to show the changes in the values of the circumference and diameter but the ratio of the circumference to the diameter of a circle remains constant as the size of the circle changes.

GSP sketch in the second lesson

In the second lesson, the second member of the LS group (Teacher 2): (1) constructed a big circle using the Compass tool and labelled its centre and the point on its circumference as O and A respectively using the Text tool and enlarged the font size of the labels from 12 to 24 using the Style option in the Label menu; (2) constructed diameter AB by (i) constructing a line using the Line tool, (ii) constructing segment AB using the Segment tool, and (iii) hide the line using the Hide Line option in the Display menu; (3) measured the circumference of the circle using the Circumference option in the Measure menu; (4) changed the label of circumference using the Label option in the Properties menu; (5) measured the diameter of the circle using the Length option in the Measure menu; (6) changed the label of diameter using the Label option in the Properties menu; (7) changed the value of the circumference and diameter to thousandths using the Value option in the Properties menu; (8) calculated the ratio of the circumference to the diameter of the circle using the Calculate option in the Measure menu; (9) changed the ratio of the circumference to the diameter of the circle to thousandths using the Value option in the Properties menu; (10) tabulated the value of the circumference, diameter and ratio of the circumference to the diameter of the circle using the Tabulate option in the Graph menu; and (11) added the values of the circumference, diameter and ratio of the circumference to the diameter of the circle by right-clicking on the table and using the Add Table Data option and selecting the Add 10 Entries As Values Change option.

Next, Teacher 2 dragged point A to gradually enlarge the circle and asked the students (peers) to observe the changes in the values of the circumference, diameter and ratio of the circumference to the diameter of the circle. The resulting GSP sketch is as shown in Figure 2. Following this, the students were asked to draw a conclusion about the value of the ratio of the circumference to the diameter of a circle based on their observations. Lastly, Teacher 2 explained that the ratio of the circumference to the diameter of a circle is known as π and its estimated value is 3.142 correct to three decimal places.

After the lesson some of the peers suggested that the diameter and circumference ought to be labelled to help them better understand the ratio of the circumference to the diameter of a circle as well as the concept of π and its estimated value. Further, the researcher suggested that the measurements of the diameter, circumference and ratio of the circumference to the diameter of the circle could be hidden so that students would only focus

on the changes in the values of the diameter, circumference and ratio of the circumference to the diameter of the circle in the table.

After the tutorial, the members of the LS group had to make final changes to their GSP sketch based on the comments and suggestions given by their peers and the researcher as well as their own observations. They also consulted the researcher on how to hide the measurements of the diameter, circumference and ratio of the circumference to the diameter of the circle.

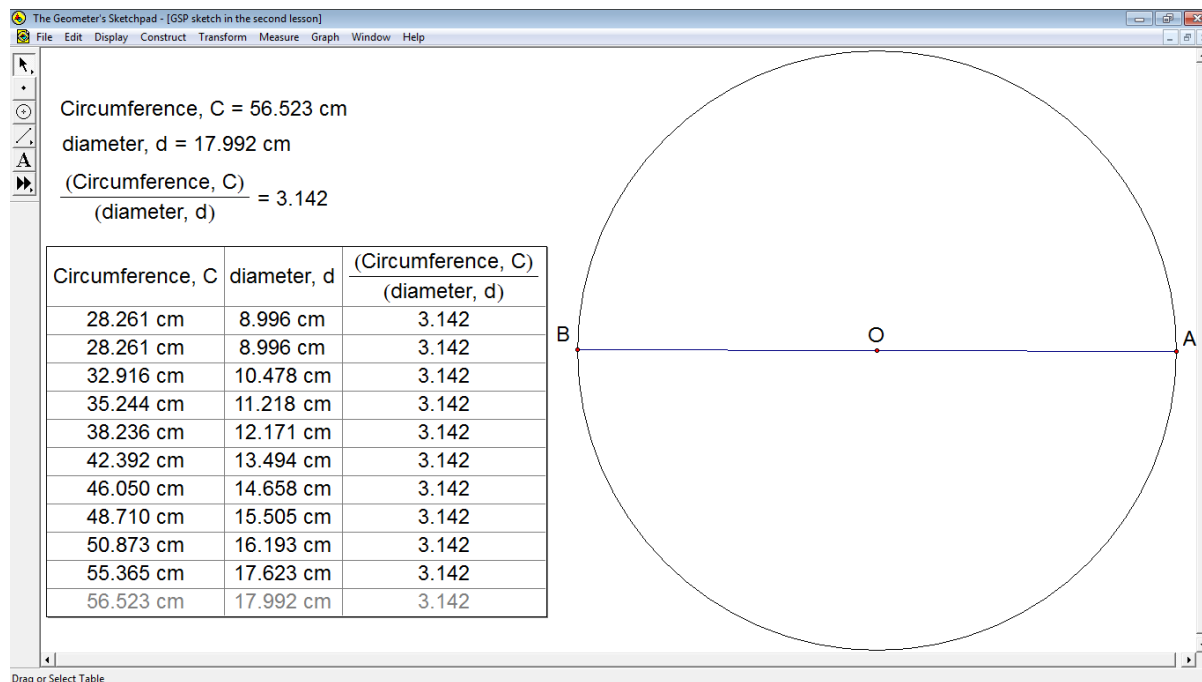


Figure 2. GSP sketch in the second lesson

GSP sketch in the third lesson

In the third lesson, as evidenced in the GSP activity sheet, the members of the LS group constructed the revised GSP sketch by following the same steps as in the second lesson (Steps 1-11). In addition, they: (1) labelled the diameter and circumference of the circle using the Text tool; (2) enlarged the font size of the labels of the diameter and circumference of the circle from 12 to 24 using the Style option in the Label menu; (3) hid the measurements of the diameter, circumference and ratio of the circumference to the diameter of the circle using the Hide Measurements option in the Display menu; (4) dragged point A to gradually enlarge the circle and asked students to observe the changes in the values of the circumference, diameter and ratio of the circumference to the diameter of the circle; (5) asked students to draw a conclusion about the ratio of the circumference to the diameter of a circle based on their observations; and (6) explained that the ratio of the circumference to the diameter of a circle is known as π and its estimated value is 3.142 correct to three decimal places. The resulting GSP sketch is as shown in Figure 3.

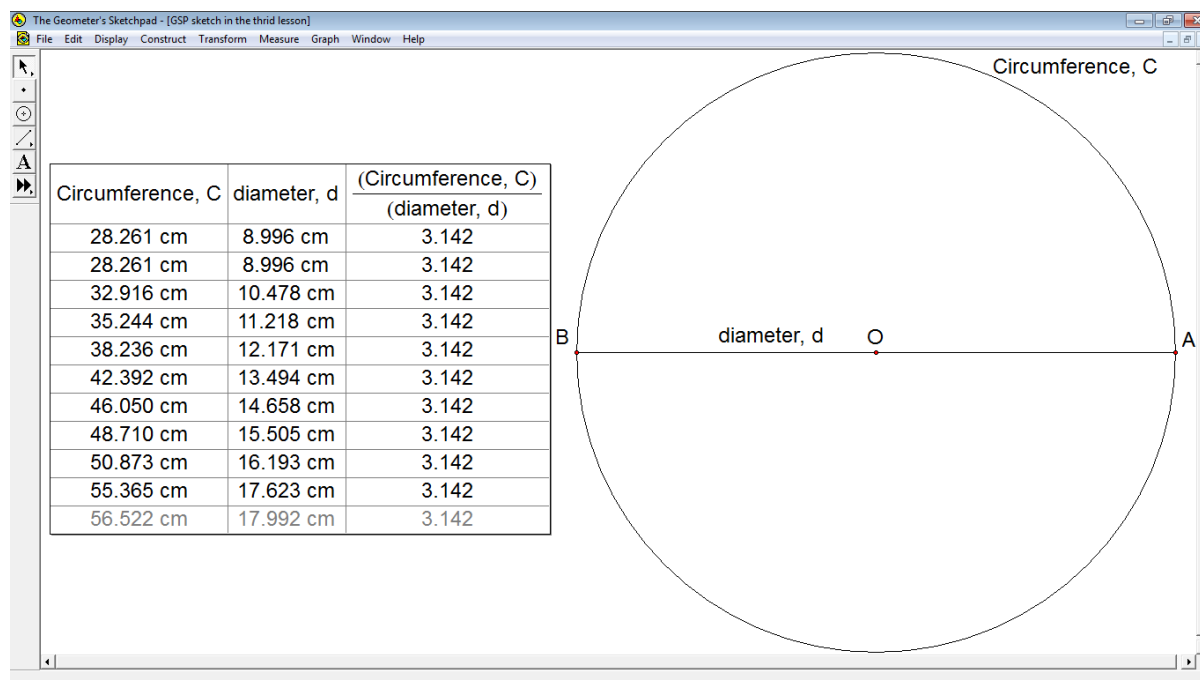


Figure 3. GSP sketch in the third lesson

Conclusion

The findings of the study from the analysis of the GSP sketches in the first, second and third lessons indicate that the participants of this LS group showed gradual improvement in their skills and confidence to construct GSP sketches for teaching the Form 2 Mathematics topic of estimating the value of π after engaging in LS. Specifically, the GSP sketches changed from single measurements of the radius, diameter, circumference and ratio of the circumference to the diameter of a circle to a table of several measurements of the radius, diameter, circumference and ratio of the circumference to the diameter of a circle. This might enable students to conclude that the ratio of the circumference to the diameter of a circle is a constant which is known as π and its estimated value is 3.142 correct to three decimal places.

The researcher acknowledges the limitations of observing the improvement in all the pre-service secondary mathematics teachers' skills and confidence to construct GSP sketches for teaching mathematics after engaging in LS. Nevertheless, the researcher was very much encouraged by the positive attitude and commitment of the participants in constructing and re-constructing the GSP sketches several times as revealed in their GSP sketches in the first, second and third lessons as well as their several consultations with the researcher.

In conclusion, LS provided an alternative means of helping the pre-service mathematics to teach mathematics using GSP in general and the topic in particular at the secondary school level.

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