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Constructing Geometer's Sketchpad Sketches for Teaching Mathematics through Lesson Study

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Abstract

The objective of this study was to develop pre-service secondary teachers' skills of constructing Geometer's Sketchpad (GSP) sketches for teaching mathematics through Lesson Study (LS). The researcher employed a case study research design and the participants comprised twenty-six pre-service secondary teachers who enrolled in a mathematics teaching methods course in a Malaysian public university. Thirteen LS groups each comprising 2 pre-service secondary teachers were set up in two tutorial groups. There were six LS groups in the first tutorial group and seven LS groups in the second tutorial group. Qualitative data were collected for each LS group through observations, written lesson plans, reflections and GSP sketches. This paper discusses the changes in the constructed GSP sketches for teaching the Form Four Mathematics topic of Gradient of a Straight Line in the Cartesian Coordinates System of one of the LS groups in the first, second and third lessons after engaging in LS. Findings of the study indicate that the participants of this LS group were able to construct a suitable and dynamic GSP sketch for teaching the topic after engaging in LS.

Keywords: Lesson Study, Geometer's Sketchpad sketches, pre-service secondary teachers, Gradient of a Straight Line

Introduction

According to Fernandez and Yoshida (2004), Lesson Study (LS) was already well established in Japan since the 1960s. It is a direct translation for the Japanese term *jugyokenkyu* (*jugyo* means lesson and *kenkyu* means study or research). By participating in LS groups, teachers actively engage in a continuous process of improving the quality of their teaching in order to enrich their students' learning experiences. LS is a process by which small groups of teachers meet at stipulated time to plan lessons, observe these lessons unfold in actual classrooms, discuss their observations and to revise the lesson plans collaboratively. In fact, a number of studies have shown that LS improves teachers' learning and supports teachers to grow professionally (Stigler & Hiebert, 1997, 1999; Shimahara, 1998; Lewis & Tsuchida, 1998; Yoshida, 1999; Lewis, 2000; Fernandez, & Yoshida, 2004; Lim, White & Chiew, 2005).

In addition, several studies have shown that LS provides worthwhile and beneficial learning experiences for pre-service teachers. For example, LS improved pre-service mathematics teachers' content knowledge and enhanced their confidence to teach mathematics, and they gained much more diverse teaching ideas that helped them improve

their pedagogical content knowledge as well (Chiew & Lim, 2003). LS also helped pre-service teachers to connect theory and practice, collaborate among themselves and reflect on their teaching and learning experiences (Fernandez & Robinson, 2006). Besides, most pre-service teachers suggested that LS was a good way of preparing them to teach mathematics and would like to continue to be involved in LS later in schools after graduation despite facing problems of time constraint and peer conflict (Lim, 2006).

Apart from that, research has also shown that LS encouraged the innovative use of GSP (a dynamic geometry software program for constructing and investigating mathematical objects) in the teaching and learning of mathematics among secondary school teachers (Chew and Lim, 2011a). The teachers showed positive changes in their knowledge and skills of using GSP to teach "Lines and Planes in Three Dimensions," "Loci in Two Dimensions" and "Plans and Elevations". They also showed positive acceptance and feedback about LS such as providing peer support and collaboration. The teachers thus had more confidence in using GSP to teach mathematics at the secondary school level after engaging in LS. Chew and Lim (2011b) also showed that LS enhanced pre-service secondary teachers' skills of using GSP to teach the Form Three Mathematics topic of 'Concept of Regular Polygons'.

Further, research has shown that GSP is an essential tool for enhancing students' learning of mathematics. For instance, mathematics achievement and time of independent investigation using GSP were significant predictors of conjecture-making ability (Elchuck,1992). The abilities to conjecture and justify conjectures in a geometry class using GSP were directly related to proof-writing abilities (Frerking, 1995). The use of GSP enhanced students' van Hiele levels of geometric thinking (Choi, 1996; Choi-Koh, 1999; July, 2001; McClintock, Jiang & July, 2002; Thompson, 2006), enhanced secondary students' geometry achievement and van Hiele levels of geometric thinking (Nurul Hidayah Lucy, 2005; Chew & Noraini Idris, 2006; Noraini Idris, 2007; Chew, 2007), enhanced primary pupils' van Hiele levels of geometric thinking of selected regular polygons (Chew & Lim, 2010), and enhanced pre-service secondary mathematics teachers' understanding of limits of sequences (Cory & Garofalo, 2011). The dynamic capability of GSP, inquiry-based tasks, as well as student-student and researcher-student interactions deepened students' conception of two-dimensional shapes (Driskell, 2004). A GSP-based courseware called 'G-Reflect' had a significant effect on secondary students' achievement and motivation in learning the topic of "Reflections" (Rosanini Mahmud, Mohd Arif Hj Ismail & Lim, 2009).

In view of its importance, the Malaysian Ministry of Education (2003) advocates the use of GSP in the teaching and learning of mathematics. However, teacher enthusiasm and willingness to use GSP in the teaching and learning of mathematics remains an issue to be addressed (Teoh & Fong, 2005). For example, a survey conducted by Kasmawati (2006) on 151 secondary mathematics teachers in Penang indicated that 26% of the teachers had attended GSP training courses but only 2% used GSP to teach mathematics in the classroom. The two main reasons given by the mathematics teachers were firstly lack of time to prepare a GSP sketch, and secondly lack of skills and confidence to use GSP to teach mathematics in the classroom. Therefore, there is an urgent need to help pre-service secondary mathematics teachers' to construct GSP sketches for teaching mathematics through a collaborative group effort such as Lesson Study which will provide helpful support and sustain the continuous use of GSP in the teaching and learning of mathematics as advocated by the Ministry of Education.

Objective of the Study

The objective of this study was to develop pre-service secondary teachers' skills of constructing Geometer's Sketchpad (GSP) sketches for teaching mathematics through Lesson Study (LS). Specifically, the aim of this paper was to examine the changes in the constructed GSP sketches for teaching the Form Four Mathematics topic of Gradient of a Straight Line in the Cartesian Coordinates System of one of the LS groups in the first, second and third lessons after engaging in LS.

Methodology

The researcher employed a case study research design and the participants comprised twenty-six pre-service secondary teachers who enrolled in a mathematics teaching methods course in a Malaysian public university. The research procedure of this study consisted of three main components.

Firstly, in the first two-hour lecture the researcher, who is the coordinator of the course, explained the outline of the course, the coursework, and the LS process to all the pre-service secondary teachers. At the end of the lecture, the pre-service secondary teachers were divided into two tutorial groups. Thirteen LS groups each comprising 2 pre-service secondary teachers were set up in the two tutorial groups. There were six LS groups (named as LS Group 1 to LS Group 6) in the first tutorial group (named as Tutorial Group A) and seven LS groups (named as LS Group 1 to LS Group 7) in the second tutorial group (named as Tutorial Group B). Each tutorial group met at a specific tutorial time for one hour every week.

Secondly, the researcher conducted two GSP workshops for each tutorial group during the first two tutorials. In the first GSP workshop which was held during the first tutorial, the participants learnt the functions of the Title bar, Menu bar, Sketch plane, and Toolbox of GSP as well as how to use the basic tools of GSP such as Selection Arrow tool, Point tool, Compass tool, Straightedge tool, Text tool, and Custom tool to construct mathematical objects like points, segments, rays, lines, circles, and polygons. In the second GSP workshop which was held during the second tutorial, the participants learnt how to construct a GSP sketch for teaching Form Two Mathematics topic of Pythagoras' Theorem based on Benett's (1999) GSP activity sheet.

Thirdly, the researcher implemented the six main steps of LS as outlined by Fernandez and Yoshida (2004). In Step 1 (Collaboratively Planning the Lesson Plan), each LS group was allowed to choose a topic in the Malaysian secondary school mathematics syllabus during the third tutorial. Then, each LS group was given time to collaboratively plan a 40-minute lesson plan for teaching the chosen topic with GSP. At the end of the tutorial, each LS group had to plan subsequent meetings outside the lecture and tutorial schedule to complete their lesson plan and GSP sketches before the fourth tutorial.

Next, in Step 2 (Seeing the Lesson Plan in Action), one participant from LS Group 1 in each tutorial group taught the 40-minute lesson as planned to their peers in the Mathematics Teaching Room during the fourth tutorial. The lesson was observed by his/her partner of LS Group 1 and the researcher. In Step 3 (Discussing the Lesson Plan), the peers and the researcher provided comments and suggestions to improve the lesson plan and GSP sketches after the lesson.

After the tutorial, that is in Step 4 (Revising the Lesson Plan), the members of LS Group 1 in each tutorial group planned subsequent meetings outside the lecture and tutorial schedule to revise their lesson plan and GSP sketches based on the comments and suggestions given by their peers and the researcher as well as their own observations before the fifth tutorial.

In Step 5 (Teaching the New Version of the Lesson), the new version of the lesson based on the revised lesson plan and GSP sketches was then taught by the other partner of LS Group 1 to different peers in the other tutorial group during the fifth tutorial. The lesson was observed by his/her partner of LS Group 1 who had taught the first lesson and the researcher. After the lesson, the peers and the coordinator provided comments and suggestions to further improve the lesson plan and GSP sketches.

Finally, in Step 6 (Sharing Reflections about the New Version of the Lesson), the members of LS Group 1 in each tutorial group planned subsequent meetings outside the lecture and tutorial schedule after the tutorial to revise their lesson plan and GSP sketches according to the comments and suggestions given by their peers and the researcher as well as their own observations before the sixth tutorial. The end product of this last step would be a final lesson plan and GSP sketches for submission to the researcher as their coursework during the sixth tutorial.

The researcher repeated Steps 2 to 6 for the other LS Groups (that is LS Groups 2, 3, 4, 5 and 6 in the first tutorial group and LS Groups 2, 3, 4, 5, 6 and 7 in the second tutorial group) in the subsequent tutorials respectively. Qualitative data were collected for each LS group through observations, written lesson plans, reflections and GSP sketches.

Findings and Discussion

In this paper, the discussion focuses on the analysis of the GSP sketches in the first, second and third lessons of one of the LS groups. This group consisted of two female pre-service secondary mathematics teachers. They chose to construct a GSP sketch for teaching the Form 4 Mathematics topic of Gradient of a Straight Line in the Cartesian Coordinates System. The changes in the participants' constructed GSP sketches in the first, second and third lessons after engaging in LS are presented and discussed in the following sections.

In the first lesson, the members of the LS group constructed a GSP sketch (see Figure 1) to enable students to understand the relationship between the value of the gradient and the direction of inclination of a straight line in the Cartesian coordinates system.

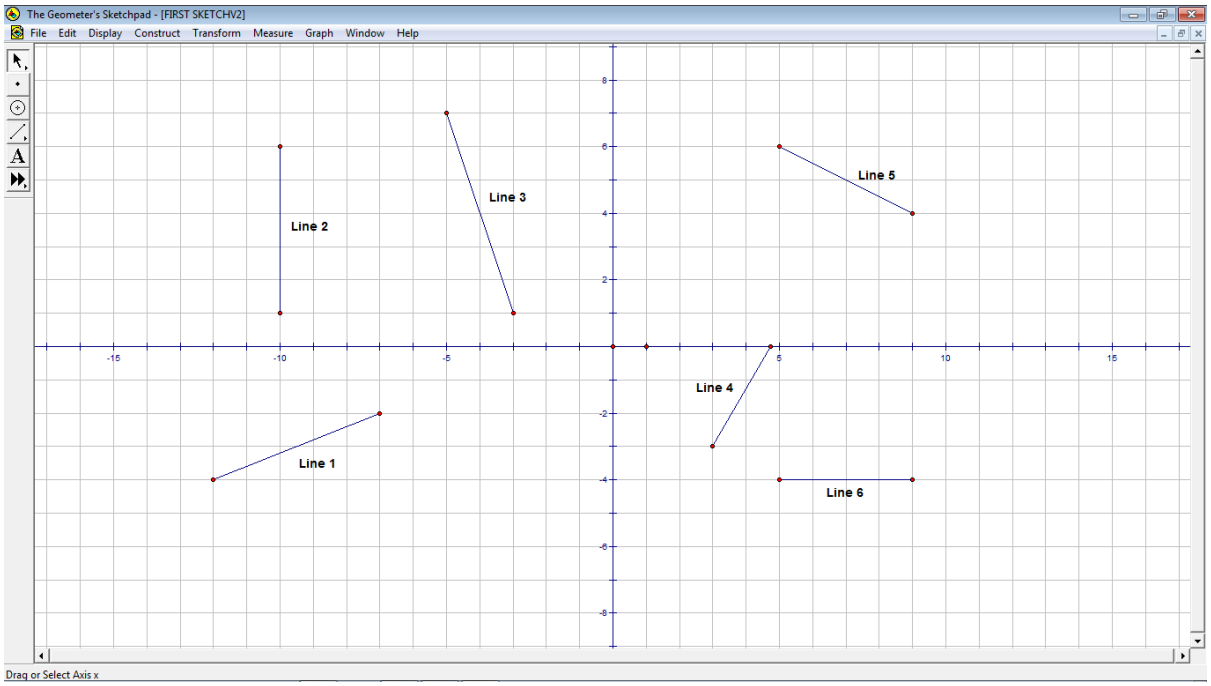

Figure 1: First GSP sketch

As shown in Figure 1, the first member of the LS group constructed six segments using the Segment tool and labelled them as Lines 1, 2, 3, 4, 5 and 6 respectively in the Cartesian coordinates system. The students were then asked to calculate the gradient of the straight lines using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$. After calculating the gradient of the straight lines, the students were asked to identify the lines that have: (i) a positive gradient, (ii) a negative gradient, (iii) a zero gradient and (iv) an undefined gradient. Finally, the students were asked to draw a conclusion about the relationship between the value of the gradient and the direction of inclination of a straight line in the Cartesian coordinates system.

After the lesson their peers generally gave positive comments about the lesson such as the lesson was interesting, the GSP sketch was easy to construct and suitable for the lesson, and the GSP sketch helped them to understand that the gradient of a straight line (i) which inclined upwards to the right is a positive value, (ii) which inclined downwards to the right is a negative value, (iii) which is parallel to the x-axis is zero, and (iv) which is parallel to the y-axis is undefined. But, they suggested that the GSP sketch ought to allow them to check their answers if possible.

The researcher generally agreed with the peers' comments, but commented that the students might misunderstood segments as lines because the constructed segments were labelled as lines in the GSP sketch (see Figure 1). The researcher clarified that segments have two endpoints while lines have no endpoints. Thus, the researcher suggested that lines ought to be constructed instead of segments to avoid misunderstanding since the topic taught was Gradient of a Straight Line in the Cartesian Coordinates System.

After the tutorial, the members of the Lesson Study group were required to make changes to their GSP sketch based on the comments and suggestions given by their peers and the researcher as well as their own observations. They were also advised to do further readings on constructing GSP sketches for teaching the topic by referring to GSP books such

as *Exploring Geometry with The Geometer's Sketchpad* (Benett, 1999) which is available in the library and other resources on the Internet.

In the second lesson, the second member of the LS group constructed six lines using the Line tool and labelled them as Lines 1, 2, 3, 4, 5 and 6 respectively in the their GSP sketch (see Figure 2). As in the first lesson, the students in the other tutorial group were asked to calculate the gradient of the straight lines using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$. After calculating the gradient of the straight lines, they were allowed to check their answers by measuring the gradient of the straight lines using the Slope command in the Measure menu of GSP. Based on the values of the gradient of the straight lines, the students were then asked to identify the lines that have: (i) a positive gradient, (ii) a negative gradient, (iii) a zero gradient and (iv) an undefined gradient. Lastly, the students were asked to draw a conclusion about the relationship between the value of the gradient and the direction of inclination of a straight line in the Cartesian coordinates system.

After the lesson their peers commented that the GSP sketch was easy to construct and suitable for the lesson as it allowed them to check their answers. They also mentioned that the GSP sketch helped them to understand the relationship between the value of the gradient and the direction of inclination of a straight line in the Cartesian coordinates system.

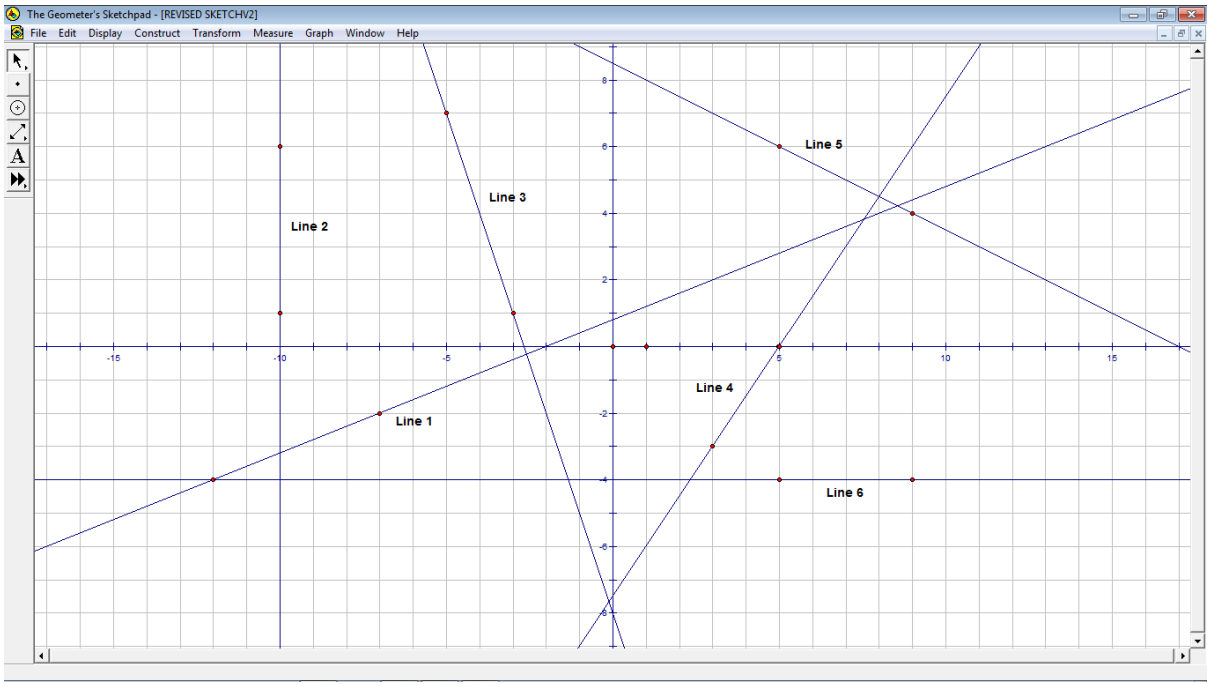

Figure 2: Revised GSP sketch

In addition, the researcher commented that although lines were constructed instead of segments, those lines did not allow students to observe the change in the values of their gradient and hence did not allow students to visualize the relationship between the value of the gradient and the direction of inclination of a straight line in the Cartesian coordinates system. Therefore, the researcher suggested that only one line ought to be constructed instead of six lines and students ought to measure the gradient of the straight line and then drag the straight line to observe the change in the values of its gradient and the direction of its

inclination in the Cartesian coordinates system. This would help students visualize the relationship between the value of the gradient and the direction of inclination of a straight line in the Cartesian coordinates system.

After the tutorial, the Lesson Study group members made further changes to their GSP sketch based on the comments and suggestions given by their peers and the researcher as well as their own observations. They also referred to GSP books and other resources on the Internet as well as sought help and guidance from the researcher. As a result, they successfully constructed the final GSP sketch as shown in Figure 3.

In the final GSP sketch, the LS group members constructed a single straight line using the Line tool and measured the gradient of the straight line using the Slope command in the Measure menu of GSP. Then, they changed the label of Slope AB to Gradient of line AB as used in the Form Four Mathematics textbook so that students could understand the meaning of the measurement. Students were asked to drag point A or B and observe the value of the gradient and the direction of inclination of the straight line. They were required to find the value of the gradient of the straight line: (i) when it inclined upwards to the right, (ii) when it inclined downwards to the right, (iii) when it is parallel to the x-axis, and (iv) when it is parallel to the y-axis. Finally, students were asked to draw a conclusion about the relationship between the value of the gradient and the direction of inclination of a straight line in the Cartesian coordinates system.

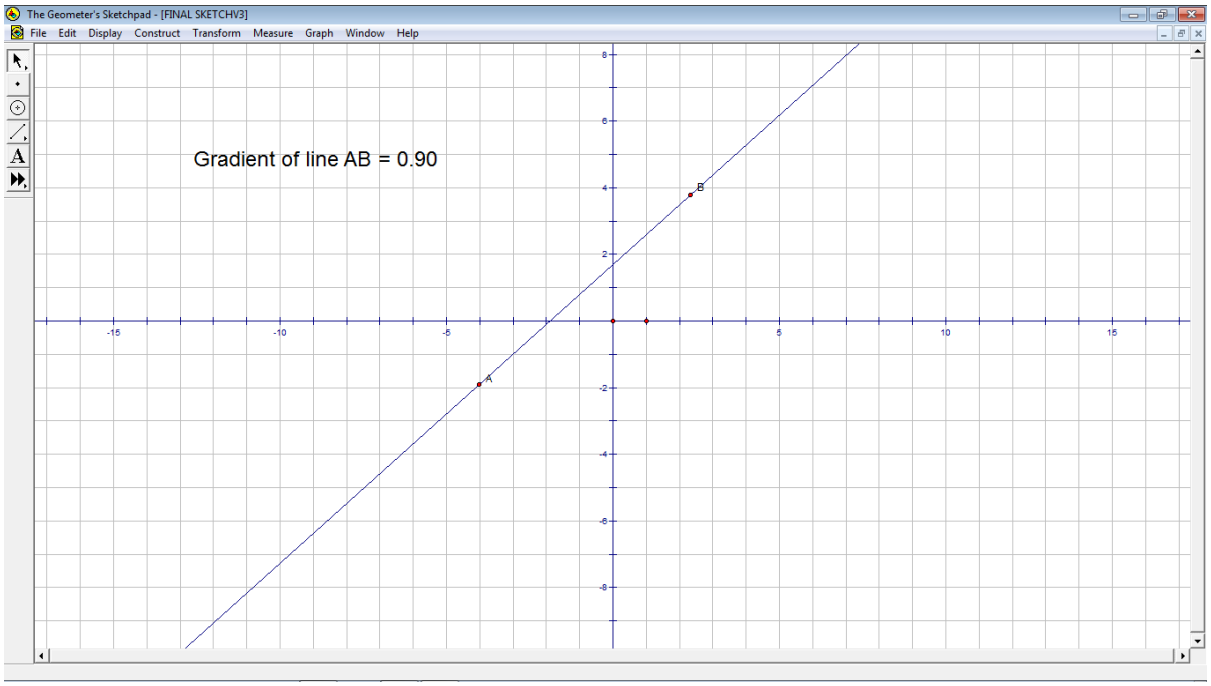

Figure 3: Final GSP sketch

Conclusion

Analysis of the GSP sketches in the first, second and third lessons indicates that the members of this LS group showed positive changes in their skills of constructing GSP sketches for teaching the Form 4 Mathematics topic of Gradient of a Straight Line in the

Cartesian Coordinates System after engaging in LS. Specifically, the GSP sketches changed from static segments and lines to a single dynamic line that enabled students to visualize the relationship between the value of the gradient and the direction of inclination of a straight line in the Cartesian coordinates system.

Initially in the first GSP sketch they constructed six segments instead of lines and students were not asked to drag any control point of the segments to observe the changes in the values of the gradient and the direction of inclination of the straight line. Although in the second GSP sketch the LS group members constructed six lines, students were also not asked to drag any control point of the segments to observe the changes in the values of the gradient and the direction of inclination of the straight line which would help them to draw a conclusion about the relationship between the value of the gradient and the direction of inclination of a straight line in the Cartesian coordinates system.

Nevertheless, in the third GSP sketch the LS group members constructed a single straight line and measured its gradient. Students were asked to dynamically drag any control point of the straight line and observe the value of the gradient and the direction of inclination of the straight line. They were asked to identify the lines that have a positive gradient, a negative gradient, a zero gradient and an undefined gradient. Lastly, students were asked to draw a conclusion about the relationship between the value of the gradient and the direction of inclination of a straight line in the Cartesian coordinates system.

The findings of this study concurred with the findings of Chew and Lim (2011b) in that LS enhanced pre-service secondary teachers' skills of constructing GSP sketches for teaching teach the Form Three Mathematics topic of 'Concept of Regular Polygons'.

In conclusion, LS provided an alternative means of improving the pre-service mathematics teachers' skills of constructing GSP sketches for teaching mathematics in general and the topic in particular at the secondary school level. Moreover, LS provided an alternative means of improving in-service secondary mathematics teachers' knowledge and skills of using GSP to teach "Lines and Planes in Three Dimensions," "Loci in Two Dimensions" and "Plans and Elevations" (Chew and Lim, 2011a).

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